

# Do we really live in four or in higher dimensions?

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In Einstein gravity, gravitational potential goes as  $1/r^{d-3}$  in  $d$  spacetime dimensions, which assumes the familiar  $1/r$  form in four dimensions. On the other hand, it goes as  $1/r^\alpha$  with  $\alpha = (d-2m-1)/m$  in *pure Lovelock gravity* involving only one  $m$ th order term of the Lovelock polynomial in the gravitational action. The latter offers a novel possibility of having  $1/r$  potential for the dimension spectrum given by  $d = 3m + 1$ . Thus it turns out that in the two prototype gravitational settings of isolated objects like black holes and the universe as a whole – cosmological models, there is *no way to distinguish* between Einstein in four and  $m$ th order pure Lovelock in  $3m + 1$  dimensions, i.e., in particular  $m = 1$  four dimensional Einstein and  $m = 2$  seven dimensional pure Gauss-Bonnet gravity. As envisaged in higher dimensional theory, all matter fields, e.g., Electromagnetic field, remain confined to the usual four dimensions while gravity is however free to propagate in higher dimensions. But it cannot distinguish between any two members of the dimension spectrum, then one wonders, do we really live in four or in higher dimensions?

*Introduction* — For any metric theory of gravity what is required is construction of a divergence free second rank symmetric tensor from the metric and curvature alone. It could be done in two ways: (a) varying action, an invariant constructed from Riemann curvature and (b) by making trace of Bianchi derivative of properly defined “Riemann tensor” vanish. For Einstein gravity, variation of Einstein-Hilbert action with respect to metric tensor or trace of Bianchi identity, both lead to Einstein’s equation [1, 2]. In order to obtain physically meaningful solutions with proper initial value problem ensuring unique evolution as well as absence of ghosts, it is imperative that equation of motion continues to remain second order as is the case for Einstein gravity. The key requirement is that it should remain so even when we go to higher orders in the Riemann tensor. This requirement uniquely singles out Lovelock polynomial action that always yields second order equation [3–7]. Lovelock action is essentially a sum of dimensionally continued Euler densities, being of order  $m$  ( $\geq 0$ ) in curvature, such that  $m = 1$  gives Einstein-Hilbert Lagrangian,  $m = 2$  Gauss-Bonnet and so on. Alternatively it is possible to define  $m$ th order Lovelock Riemann tensor [8–10] and trace of its Bianchi derivative yields a divergence free second rank symmetric tensor, an analogue of Einstein tensor, as obtained from variation of  $m$ th order Lovelock action.

Lanczos-Lovelock gravity is important for various reasons. It is generally believed that Einstein-Hilbert action is an effective action, valid at small enough energy (or large length) scales. At high energy near the Planck scale gravity cannot be described by Einstein-Hilbert action alone but should be supplemented by higher curvature terms. As requirement of the field equations being second order uniquely singles out Lanczos-Lovelock Lagrangian from all others. For example, supersymmetric string the-

ory *exactly* reproduces Gauss-Bonnet term as quadratic correction to Einstein-Hilbert action [11]. Further thermodynamic interpretation of Einstein’s equation generalizes in a straightforward and natural *but* non-trivial manner to Lanczos-Lovelock gravity. The same is true for other correspondences between thermodynamics and gravity in the context of general relativity as well [12–16]. Finally pure Lovelock theories, i.e., a single term in Lovelock polynomial, exhibit very interesting features. These include — (a) there is a close connection between pure Lovelock and dimensionally continued black holes [17], (b) gravity is kinematic in all critical  $d = 2m + 1$  dimensions, i.e., vacuum is pure Lovelock flat [18], (c) bound orbits exist for a given  $m$  in all  $2m + 1 < d < 4m + 1$  dimensions, in contrast for Einstein gravity they do so only in four dimension [17] and finally (d) equipartition of gravitational and non-gravitational energy defines location of black hole horizon [19, 20].

To these interesting features we wish to add in this letter yet *another remarkable property* of pure Lovelock gravity. In Einstein gravity potential goes as  $1/r^{d-3}$  that takes the familiar  $1/r$  form only in four dimension and none else. In contrast, for pure Lovelock it goes as  $1/r^\alpha$  with  $\alpha = (d - 2m - 1)/m$ , and hence there exists dimension spectrum  $d = 3m + 1$  for  $1/r$  potential. This can happen only in pure Lovelock gravity and hence this is also a very important distinguishing property for pure Lovelock theories. What it means is that there is *no way* to fathom from solar system observations whether it is four dimensional Einstein or seven dimensional pure Gauss-Bonnet or in general  $(3m + 1)$ -dimensional pure  $m$ th order Lovelock gravity. Further it also turns out that the standard FLRW cosmology remains the same in the dimension spectrum.

In higher dimensional theories it is envisaged that all matter fields remain confined to the usual four dimension (termed as 3-brane) while gravity is however free to propagate in higher dimensions. Since gravity remains the same for the entire dimensional spectrum  $d = 3m + 1$ , there is no way to distinguish between any two members

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of the spectrum. In particular we may as well be living in seven dimensional spacetime rather than the usual four with gravity being described by pure Gauss-Bonnet instead of Einstein gravity. This is the question we wish to pose and wonder about in this letter.

*Lovelock Gravity* — In a  $d$ -dimensional spacetime, gravity in general, can be described by an action func-

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$$L = \sum_m c_m L_m = \sum_m c_m \frac{1}{2^m} \delta_{c_1 d_1 c_2 d_2 \dots c_m d_m}^{a_1 b_1 a_2 b_2 \dots a_m b_m} R_{a_1 b_1}^{c_1 d_1} R_{a_2 b_2}^{c_2 d_2} \dots R_{a_m b_m}^{c_m d_m} \quad (1)$$


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where  $\delta_{rs\dots}^{pq\dots}$  stands for completely antisymmetric determinant tensor. The case  $m = 2$  is Gauss-Bonnet Lagrangian which is quadratic in curvature and reads as  $L_{GB} = (1/2)(R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2)$ . Remarkably it also appears in the low energy effective theory of supersymmetric strings, as first pointed out in [11]. Further Lovelock Lagrangian is a sum over  $m$  with each term is a homogeneous polynomial in curvature and has a dimensionful coupling constant. Further complete antisymmetry of the determinant tensor demands  $d \geq 2m$  else it would vanish identically. Even for  $d = 2m$  Lanczos-Lovelock Lagrangian reduces to total derivative – pure surface term. Lovelock Lagrangian  $L_m$  is therefore non-trivial only in dimension  $d \geq 2m + 1$ . Note that pure Lovelock gravity is kinematic in all critical odd  $d = 2m + 1$  dimensions [10, 18] because  $m$ th order Riemann is entirely given in terms of corresponding Ricci hence it has non non-trivial vacuum solution. Thus non-trivial vacuum solutions only exist in dimensions  $d \geq 2m + 2$ . Finally, variation of the Lagrangian for pure Lovelock theories lead to the following second order equation,

$${}^{(m)}E_b^a \equiv -\frac{1}{2^{m+1}} \delta_{ba_1 b_1 \dots a_m b_m}^{ac_1 d_1 \dots c_m d_m} R_{c_1 d_1}^{a_1 b_1} \dots R_{c_m d_m}^{a_m b_m} = 8\pi T_b^a \quad (2)$$

Since there appears no derivatives of curvature, hence the equation is as expected of second order. Though not directly visible, second derivative too appears linearly; i.e., it is quasilinear thereby ensuring unique evolution.

*Black Holes* — Let us now consider pure Lovelock vacuum spacetime and write spherically symmetric metric in Schwarzschild gauge incorporating the null energy condition, as given by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2 \quad (3)$$

where  $f(r) = 1 + 2\Phi(r)$  and  $d\Omega_{d-2}^2$  is metric on  $(d-2)$ -sphere [21]. Given this metric ansatz, the (t,t) component of the equation,  ${}^{(m)}E_t^t = 0$  could be trivially integrated as it takes the simple form  $(r^{d-2m-1}\Phi^m)' = 0$

tional involving arbitrary scalar functions of metric and curvature constructed out of the metric, but not of its derivative. In general, variation of this arbitrary Lagrangian would lead to an equation having fourth order derivatives of metric. For it to be of second order, gravitational Lagrangian is constrained to be of the following Lovelock form,

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where prime is derivative relative to  $r$ , and so we obtain

$$\Phi = -\frac{M}{r^\alpha}, \quad \alpha = (d - 2m - 1)/m \quad (4)$$

where  $M$  is a constant of integration which identifies to  $m$ th root of the ADM mass [4]. In general for static spherically symmetric metric in Schwarzschild gauge, equation  ${}^{(m)}E_t^t = 0$  is first integral of  ${}^{(m)}E_\theta^\theta = 0$ , and so we have complete vacuum solution for pure Lovelock static black hole.

Clearly there is no way to distinguish between four dimensional Schwarzschild black hole and  $(3m + 1)$  dimensional  $m$ th order pure Lovelock black hole because potential goes as  $1/r$  for all of them. As a matter of fact dimensional spectrum is given by  $d = (2 + \alpha)m + 1$  corresponding to a given value of  $\alpha$ . The choice  $\alpha = 2$  at the first level corresponds to five dimensional Einstein with dimensional spectrum  $d = 4m + 1$  while  $\alpha = 1/2$  for six dimensional Gauss-Bonnet with dimensional spectrum  $d = 5m/2 + 1$ . Also note that since horizon structure and potential are the same, all properties associated with horizon, e.g., Hawking radiation, should remain the same. Another test that can distinguish these two black holes corresponds to *black hole entropy*. But remarkably in the case under consideration, entropy density of the black hole goes as product of  $(m - 1)$  curvature, each of which scales as  $r^{-3(m-1)}$  for  $d = 3m + 1$ , while area element scales as  $r^{3m-1}$  and hence entropy would always scale as  $r_h^2$ , where  $r_h$  is horizon radius irrespective of Lovelock order  $m$  (see, however the case for geometry quantized theories in [22]). For pure Lovelock static black holes, it has been shown that thermodynamical parameters, temperature and entropy bear the same relation to horizon radius in all critical odd and even  $d = 2m + 1, 2m + 2$  dimensions [23].

The next question arises, would black hole, like Schwarzschild in four dimension, be stable in higher dimensions? In stability analysis one decomposes metric perturbations into three types, tensor, vector and scalar perturbations [24–29]. Stability analysis of Lovelock gravity black holes under scalar, vector and tensor perturbations was carried out in [30–34], which was further applied to pure Lovelock black holes in [35]. Under

metric perturbation, Riemann tensor and hence all geometric constructs will be perturbed. Using which one can obtain a differential master equation for perturbation, which is like Schrödinger equation with a potential. Only when corresponding differential operator is a positive (or zero) self-adjoint operator, solutions will be perturbatively stable if and only if there are no negative eigenvalues. This in turn leads to the following condition

$$\frac{d - 3m - 1}{mr} \geq 0 \quad (5)$$

Clearly it would be stable for  $d \geq 3m + 1$  which means  $\alpha = (d - 2m - 1/m) \geq 1$ . This is another *very interesting and novel* feature of our work. The above result explicitly shows that Einstein gravity in four or higher dimensions is always stable, since for Einstein gravity, one has  $\alpha \geq 1$  while it is unstable for pure Lovelock gravity for which  $\alpha = 1/m < 1$ . Thus the determining factor is  $\alpha \geq 1$ . Since it is stable under tensor perturbations and hence it would be so for vector perturbations as well.

*FLRW Cosmology* — From black hole we now come to cosmology and ask the question, would the standard FLRW cosmology remain the same for the dimensional spectrum  $d = 3m + 1$ ? That means this feature is not only restricted to vacuum solutions but is true for cosmology as well. For that we consider the standard FLRW metric for homogeneous and isotropic universe,

$$ds^2 = -dt^2 + a^2(t) [dr^2 + r^2 d\Omega_{d-2}^2] \quad (6)$$

From Eq. (2) we obtain for Hubble parameter,  $H = \dot{a}/a$  as,

$$H^{2m} = \frac{(d - 2m - 1)!}{(d - 1)!} 16\pi G\rho \quad (7)$$

and isotropic pressure

$$\frac{1}{2} \frac{(d - 2)!}{(d - 2m - 1)!} H^{2m-2} \times \left[ 2m \frac{\ddot{a}}{a} + (d - 2m - 1)H^2 \right] = -8\pi Gp \quad (8)$$

Deriving the conservation equation from Eq. (7) and Eq. (8) is straightforward, one needs to take time derivatives of Eq. (7) and then use Eq. (8) to eliminate  $\dot{H}$  term, resulting in  $\dot{\rho} + (d - 1)(\rho + p)H = 0$ . Assuming an equation of state  $p = \omega\rho$  with a constant equation of state parameter  $\omega$  we obtain time evolution of scale factor as  $a(t) \sim t^{2m/[(d-1)(1+\omega)]}$ . It is immediate that for  $d = 3m + 1$ , one obtains  $a(t) \sim t^{2/[3(1+\omega)]}$ , indistinguishable from the standard general relativistic cosmology.

Thus cosmological dynamics is the same in all members of the dimensional spectrum,  $d = 3m + 1$ . That is, our Universe may be four dimensional with Einstein gravity or seven dimensional with GB gravity, and there is no way to break this dilemma.

*Discussion* — There is a very strong and compelling observational evidence for  $1/r$  potential in four dimensions in the framework of general relativity which is a metric theory of gravity. It is curious to ask, could gravitational potential have the same behavior in any other dimension in some other metric theory? It is remarkable that the answer is yes, and it is uniquely the pure Lovelock theory that offers a spectrum of dimensions,  $d = 3m + 1$  for which potential is indeed  $1/r$ . This means all observations involving stellar objects and black holes including physical phenomena around them will not be able to distinguish between any two members of this spectrum. Not only that cosmological dynamics is also driven by the same inverse square law and hence it is expected that should also remain the same. And that indeed is the case as we have just shown above.

This explicitly demonstrates that the two most fundamental problems for a gravitational theory, compact objects and cosmology have the same dynamics for this dimensional spectrum. That is, all astrophysical phenomena involving accretion and galactic dynamics as well as structure formation and cosmological evolution and dynamics would be the same for dimension spectrum. This is a very important conclusion that simply follows from pure Lovelock gravity. In particular, purely based on gravitational observations it would be impossible to decipher whether it is the usual four dimensional spacetime with Einstein gravity or seven dimensional spacetime with pure Gauss-Bonnet gravity. According to string theory, it is believed that all matter fields remain confined to 3-brane or four dimensional spacetime while gravity is however free to be anywhere and not confined to four dimensions. Since gravitational dynamics cannot distinguish between any two members of the spectrum, it is a pertinent question to pose, do we really live in four dimensions or in higher dimensions with gravity given by appropriate Lovelock order? One might however ask why are higher dimensions not visible? All matter fields, e.g., electromagnetic field remain confined to 3-brane and hence they cannot probe higher dimensions. Gravity can but that can't distinguish between any two members of the spectrum. There seems no way to break this degeneracy. Thus we could wonder, do we really live in four or seven dimensional spacetime, for the latter gravity being described not by Einstein but by pure Gauss-Bonnet theory?

Gravity is abundant in complexity and richness of structure that one is never disappointed as one probes deeper and wider. The present question offers an excellent example of this richness as it coaxes one to wonder, is the universe four dimensional or not? This is a profound question that would have enormous scientific as well as philosophical fallout.

There still remains one consideration that may perhaps turn out discriminator. It is propagation of gravitational wave and its number of polarizations and degrees of freedom. It had been argued that number of degrees of freedom remain the same as given by  $d(d - 3)/2$  for Ein-

stein as well as Lovelock theory [36], and it has recently been further reinforced for pure Lovelock gravity [37]. In that case two polarizations for gravitational wave could only occur in four dimension and none else. Even then it is very intriguing that even though gravitational law remains the same – inverse square, yet number of polarizations of gravitational wave or degrees of freedom would be different in two members of the spectrum. This is a very involved and complicated question that needs further and deeper investigation, which we plan to present in a future publication. The details of this study would be given elsewhere.

We thus conclude that it is impossible to decipher, except for number of degrees of freedom, between any two members of the dimensional spectrum, raising the question, “Do we really live in four or higher dimensions?”.

It perhaps points to the fact that gravity is abundant in complexity and richness of structure that one is never disappointed as one probes deeper and wider. The present question offers an excellent example of this richness as it coaxes one to wonder, is the universe four dimensional or not? This is a profound question that would have enormous scientific as well as philosophical fallout.

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